Ivan Franko National University of Lviv

Pidstryhach Institute for Applied Problems of Mechanics and Mathematics of the National Academy of Sciences of Ukraine

National University "Lviv Polytechnic"

International Conference on

DIFFERENTIAL EQUATIONS

Dedicated to the 110th Anniversary of Ya. B. Lopatynsky

BOOK OF ABSTRACTS

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EXISTENCE OF ENTROPY SOLUTIONS FOR NONLINEAR ELLIPTIC DEGENERATE ANISOTROPIC EQUATIONS

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Let Ω be a bounded domain in $\mathbb{R}^n$ with a boundary $\partial\Omega$, $n \geqslant 2$. Assume that $1 < q_i < n$ are real numbers, and ν_i are nonnegative functions in Ω such that $\nu_i > 0$ a.e. in Ω, $\nu_i \in L^1_{\mathrm{loc}}(\Omega)$, $\nu_i^{-1/(q_i-1)} \in L^1(\Omega)$, $i = 1, \dots, n$. Suppose the Carathéodory functions $a_i : \Omega \times \mathbb{R}^n \to \mathbb{R}$, $i = 1, \dots, n$, satisfy the conditions of growth, strict monotonicity and the following coercitivity condition: $\sum\limits_{i=1}^{n} a_i(x,\xi)\xi_i \geqslant c\sum\limits_{i=1}^{n} \nu_i|\xi_i|^{q_i} - g(x)$; here c is a positive constant, and $g \in L^1(\Omega)$ is a nonnegative function. Set $q = \{q_1, \dots, q_n\}$, $\nu = \{\nu_1, \dots, \nu_n\}$. We define $W^{1,q}(\nu,\Omega) = \{u \in L^1(\Omega) : \nu_i|D_i u|^{q_i} \in L^1(\Omega), i = 1, \dots, n\}$. $W^{1,q}(\nu,\Omega)$ is a Banach space with respect to the norm

$$\|u\|_{W^{1,q}(\nu,\Omega)} = \|u\|_{L^1(\Omega)} + \sum_{i=1}^{n}\Big(\int_\Omega \nu_i|D_i u|^{q_i}\,dx\Big)^{1/q_i}.$$

Denote by $\mathring{W}^{1,q}(\nu,\Omega)$ the closure of $C_0^\infty(\Omega)$ in $W^{1,q}(\nu,\Omega)$. Let $F : \Omega \times \mathbb{R} \to \mathbb{R}$ be a Carathéodory function. We consider the Dirichlet problem

$$-\sum_{i=1}^{n} \frac{\partial}{\partial x_i} a_i(x,\nabla u) = F(x,u) \quad \text{in} \quad \Omega, \tag{1}$$

$$u = 0 \quad \text{on} \quad \partial\Omega. \tag{2}$$

Definition. *An entropy solution of problem (1), (2) is a function* $u : \Omega \to \mathbb{R}$ such that: 1) $T_k(u) \in \mathring{W}^{1,q}(\nu,\Omega)$, where T_k is a standard cut function of the level $k > 0$;
2) $F(x,u) \in L^1(\Omega)$;
3) if $w \in \mathring{W}^{1,q}(\nu,\Omega) \cap L^\infty(\Omega)$, $k \geqslant 1$, and $l \geqslant k + \|w\|_{L^\infty(\Omega)}$, then

$$\int_\Omega \Big\{\sum_{i=1}^{n} a_i(x,\nabla T_l(u)) D_i T_k(u-w)\Big\}\,dx \leqslant \int_\Omega F(x,u)\,T_k(u-w)\,dx.$$

Theorem. *Suppose the following conditions are satisfied: (i) for a.e. $x \in \Omega$ the function $F(x,\cdot)$ is nonincreasing on $\mathbb{R}$; (ii) for any $s \in \mathbb{R}$ the function $F(\cdot,s)$ belongs to $L^1(\Omega)$. Then there exists an entropy solution of problem (1), (2).*

Proof is based on the results represented in [1], [2].

1. Kovalevsky A.A., Gorban Yu.S. *On T-solutions of degenerate anisotropic elliptic variational inequalities with L^1-data*, Izv. Math. 75 (1) (2011) 101-160.
2. Kovalevsky A.A. *Entropy solutions of Dirichlet problem for a class of nonlinear elliptic fourth order equations with L^1-right-hand sides*, Izv. Math. 65 (2) (2001) 27-80.