Mathematical Model of Volume Optical Interconnection in $LiNbO_3$ Crystal

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Abstract—**The mathematical model of volume optical interconnection elements that are formed within the LiNbO3 crystal is proposed in the paper. The simulations using this model were conducted. The possibility to reconfigure the structure of optical interconnections was demonstrated.**

Index Terms—**Acousto-optic; computing environment; photorefractive media**

I. Introduction

Optical interconnections with big bandwidth and low energy consumption [1] are widely used in advanced computer systems [2]. Optoelectronic high-speed and high-performance arithmetic-logic units and data stream processors often contain matrix of optical switches. It is a matrix of diffractive optical elements (DOE) [3] or it is an array of spatial light modulators [4], respectively. Both of these two functions can be done by acousto-optic modulators, which can form a matrix of volume interconnections [5] with a structure formed by phase relations between control signals of electronic preprocessing units [6].

It is very promising to make a DOE as an array of diffraction gratings in photorefractive crystal. It allows to store operands directly in a material medium in which they will be processed [7, 8]. Changes in interconnections structure of optoelectronic processor which appear during DOE overwriting can be considered as dynamic change of processor structure [9]. Development of a method of optic computing devices design and calculation of their parameters requires simulations and modeling of physical processes [10] including knowledge about electromagnetic wave propagation in photorefractive reversible storage media. Preliminary results of mathematical modeling of recording/reading processes based on simplified methods of signal theory are need to be taken into account [11].

We develop mathematical model of interconnection pattern formed in photorefractive crystal and consider the problem of changes of structure in volume interconnections in optical computing devises with a dynamical architecture.

II. Mathematical Model of Volume Optical Interconnection

Let us consider the system of two acousto-optical modulators (AOM) 1 and 2 which are Bragg angle θ_B rotated about axis x, as it is shown in the Fig.1. Optical beam with a Gaussian distribution of intensity $E_{in}(x,z,t)$ impinges on them:

$$E_{in}(x,z,t) = E_G \exp\{j(\omega t - kz)\},$$

where $E_G = E_0 \exp\{-x^2/w_0^2\}$, E_0 - electric field intensity, w_0 - waist of the laser beam, ω - angular frequency, $k = 2\pi/\lambda$ - wave vector, $\lambda = 0.532$ mcm – wavelength in a free space.

Diffraction component of the 1st order that corresponds to transmission coefficient of acousto-optic cell in Bragg mode [11, 12] and direction of acoustic wave propagation is formed at the output of each modulator AOM 1 and AOM 2.

$$\begin{aligned} &E_{AOM1}(x,z,t) = \\ &= \eta_1 E_{G1}(x,z)\exp\{j(\omega t - kz)\}\exp\{j(Kx - \Omega t)\}, \end{aligned} \tag{1}$$

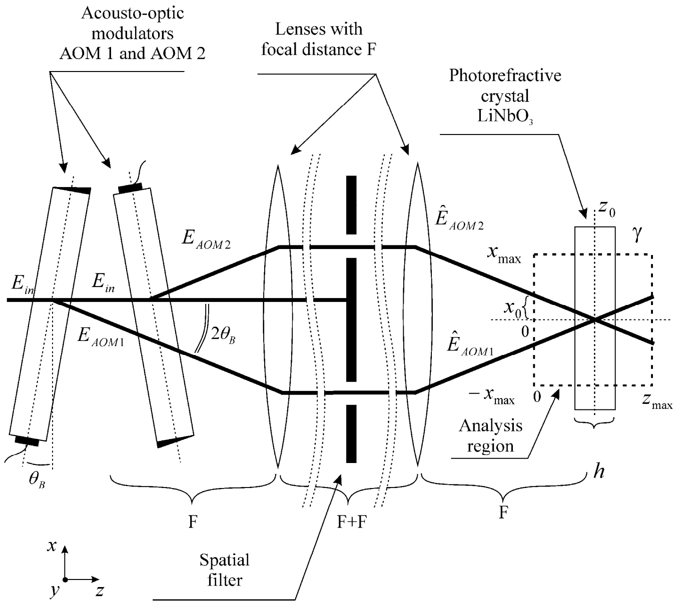

Figure 1. Optical setup for interconnection recording

$$E_{AOM2}(x,z,t)= \\ =\eta_2 E_{G2}(x,z)\exp\{j(\omega t-kz)\}\exp\{j(-Kx-\Omega t)\}, \quad (2)$$

where η_1, η_2 stands for coefficients that include the 1st order diffraction efficiency in AOM 1 and AOM 2, respectively,

$$E_{G1}(x,z)=E_0\exp\left\{-(x\cos 2\theta_B+z\sin 2\theta_B)^2/w_0^2\right\},$$

$$E_{G2}(x,z)=E_0\exp\left\{-(x\cos 2\theta_B-z\sin 2\theta_B)^2/w_0^2\right\},$$

θ_B - Bragg angle, $K=2\pi/\Lambda$ - acoustic wave vector, Λ and Ω - wavelength and angular frequency of an acoustic wave.

Expressions for diffraction components $\hat{E}_{AOM1}$ and $\hat{E}_{AOM2}$ after passing through the system of lenses can be obtained in similar to (1, 2) way in considered range (Fig.1) taking into account changes on direction of their propagation. Thus, $\hat{E}_{G1}(x,z)$ and $\hat{E}_{G2}(x,z)$ will be the following:

$$\hat{E}_{G1}(x,z)=E_0\exp\left\{-[(x+x_0)\cos 2\theta_B-z\sin 2\theta_B]^2/w_0^2\right\},$$

$$\hat{E}_{G2}(x,z)=E_0\exp\left\{-[(x-x_0)\cos 2\theta_B+z\sin 2\theta_B]^2/w_0^2\right\}.$$

In stationary mode according to (1,2) distribution of refractive index in photorefractive crystal caused by simultaneous impact of beam $\hat{E}_{AOM1}$ and $\hat{E}_{AOM2}$ can be expressed as [13, 14]:

$$n(x,z)=n_e+n_2\left|\hat{E}_{AOM1}+\hat{E}_{AOM2}\right|^2=n_e+ \\ +n_2\left(\eta_1^2\hat{E}_{G1}^2+\eta_2^2\hat{E}_{G2}^2+2\eta_1\eta_2\hat{E}_{G1}\hat{E}_{G2}\cos 2Kx\right), \quad (3)$$

where n_e is linear refractive index of extraordinary wave; n_2 is effective nonlinear refractive index. Expression (3) with $z_0-h/2\le z\le z_0+h/2$ (Fig. 1) presents mathematical model of interconnection pattern formed in photo refractive crystal.

III. The Modelling of Switching Properties of Optical Interconnection Matrix

According to Fig.1 interconnection pattern is formed as a result of recording of diffraction gratings in photorefractive crystal. Spatial distribution of diffraction peaks changes depending on the properties of diffraction grating. This fact determines structure of spatial interconnections [3].

In practice it is possible to create a matrix of optical switchers that work in Raman-Nath mode under non-modulated laser beam or more complicated architectures of optoelectronic computing devices using acousto-optic modulators and photorefractive reversible storage media [15] which can form correlation type acousto-optic processor.

It is advisable to perform modeling of interconnection pattern forming process with a help of propagating beam method [16] because of the enough wide analyzed area in comparison with light wavelength. In the case of harmonic monochromatic wave solution of the wave equation can be found in form of $\mathbf{E}(\mathbf{r},t)=\mathbf{E}(\mathbf{r})\exp(j\omega t)$. It leads to the next equation for $\mathbf{E}(\mathbf{r})$:

$$\nabla\times(\nabla\times\mathbf{E})-k_0^2n^2\mathbf{E}=0, \quad (4)$$

where k_0 is a wave number in free space, n is refractive index. Solution of (4) can be expressed in form of slowly changing envelope $\mathbf{E}_{env}(\mathbf{r})$ and fast oscillating phase coefficient:

$$\mathbf{E}(\mathbf{r})=\mathbf{E}_{env}(\mathbf{r})\exp(-j\mathbf{k}\cdot\mathbf{r}).$$

After a substitution of the last expression in (4) one can obtain the next relation for $\mathbf{E}_{env}(\mathbf{r})$:

$$(\nabla-j\mathbf{k})\times((\nabla-j\mathbf{k})\times\mathbf{E}_{env})-k_0^2n^2\mathbf{E}_{env}=0. \quad (5)$$

It is necessary to add boundary conditions for electric field $\mathbf{E}_{env}(\mathbf{r})$ at borders of the region γ under consideration. These borders are planes with $z=0$, $z=z_{\max}$, $x=-x_{\max}$, $x=x_{\max}$ (Fig. 1). We chose condition of border "transparency" for plane waves. It means no reflection at the borders:

$$\mathbf{n}\times(\nabla\times\mathbf{E}_{env})-(jk)\mathbf{n}\times(\mathbf{E}_{env}\times\mathbf{n})= \\ =-\mathbf{n}\times\{\mathbf{E}_0\times[(ik)\mathbf{n}-i\mathbf{k}]\}\exp(-j\mathbf{k}\cdot\mathbf{r}), \quad (6)$$

here $\mathbf{n}$ is a unit vector of outer normal to the border, $\mathbf{E}_0$ is electric field intensity of incident wave. Acousto-optical medium is considered homogeneous along the y axis.

Modelling of recording process can be done with a considering the fact that incident light beams have magnetic field components directed only along y axis while electric components lay in xz plane at the angle of $\pm 2\theta_B$ to z axis and have Gaussian distribution of magnitude. Size of modeled region is 30×150 mcm^2 for photorefractive storage media with refractive index $n=2{,}2$, wavelength of light is z, size of the analyzed media is $124\lambda\times 620\lambda$, where λ is wavelength of light in the media under consideration.

Modeling of switching properties of interconnection pattern was done with an assumption that incident light beam also has magnetic components directed only along y axis, electric components supposed to be in xz plane, distribution of magnitude is Gaussian, but incident light beam was perpendicular to recorded diffraction grating in the crystal (Raman-Nath diffraction). We investigated diffraction on the gratings with the period equal to 1um, 1.5um and 2um and with the size of modeled region equal to $100 \times 150\ \text{um}^2$.

Modeling was done with Comsol Multiphysics package using Beam Envelope method. We developed models of interacting regions. Results of modeling of interconnection pattern formation process are given in the Fig. 2, 3.

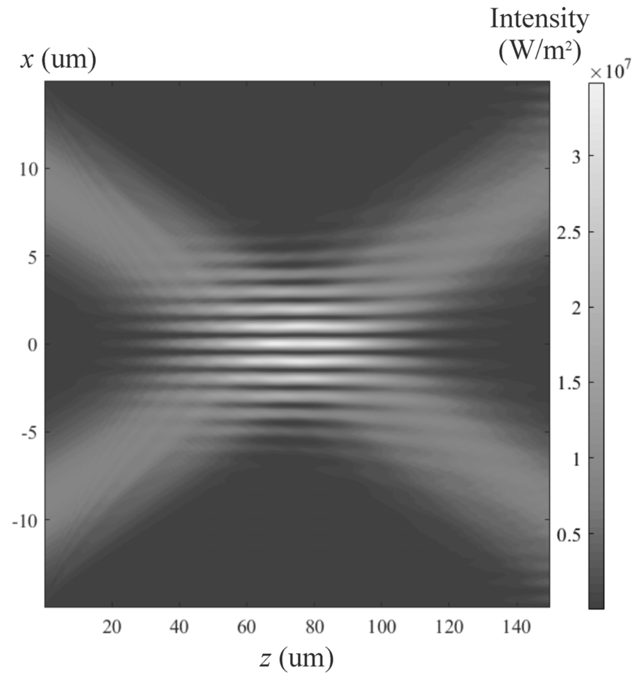

Figure 2. Spatial distribution of light intensity within the analysis region

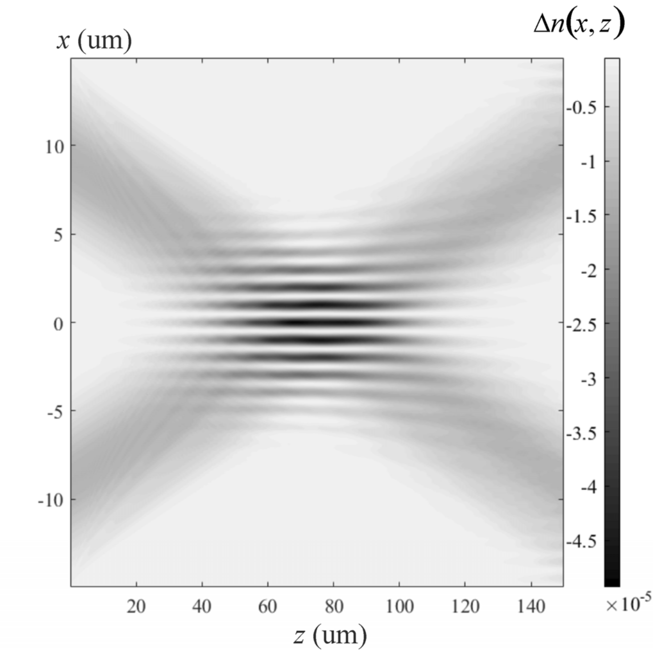

Figure 3. Refraction index distribution

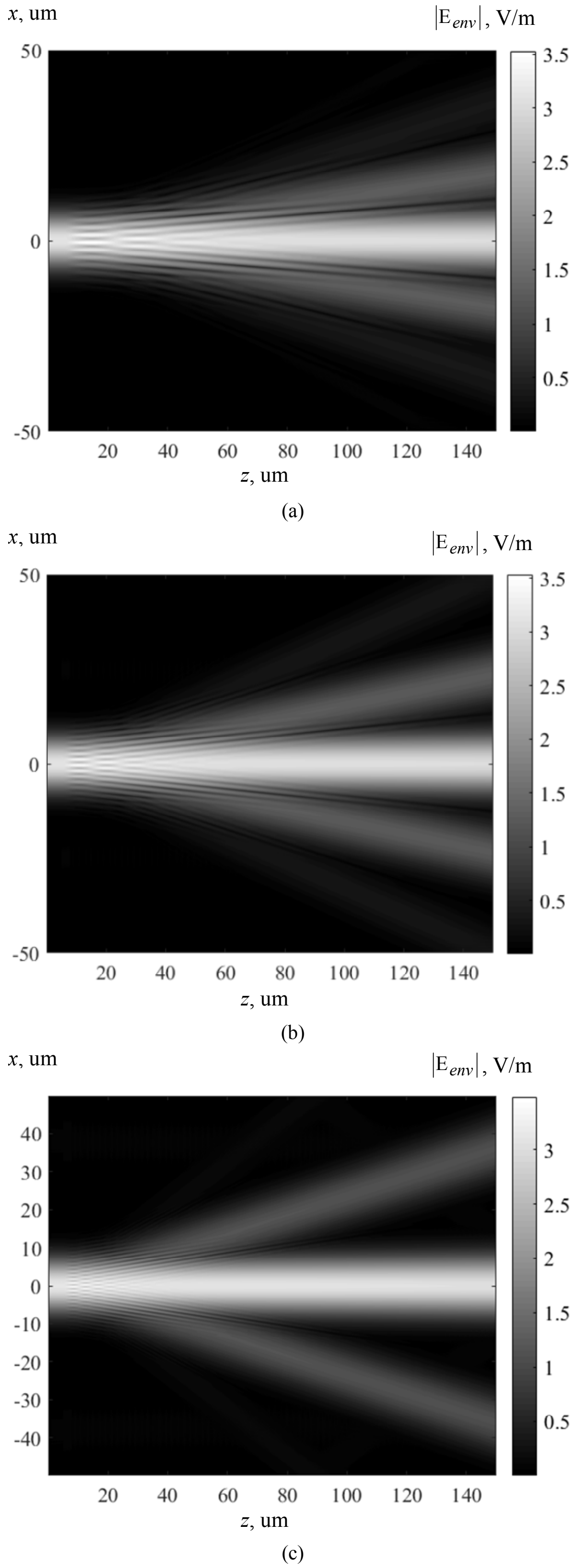

Figure 4. Illustration of switching direction change for the diffracted light, that depends on pettern structure

Fig.2 shows spatial distribution of light intensity in analyzed area. One can see changing of light beam intensity with a period of synthesized diffraction grating equal to half-period of an acoustic wave in the medium on the enlarged fragment. Geometrical size of this area is determined by beam waist and convergence angle of the laser beams, i.e. it is possible to form gratings for different modes of diffraction.

Fig.3 shows refraction index changes caused by light wave in photorefractive crystal of lithium niobate crystal.

Diffraction patterns corresponding to different interconnection patterns with different switching properties are shown in the fig.4.

Characteristic size of the region where interconnection pattern for the 0.532mcm wavelength of light is localized is equal to 100x100 mcm that is connected with diffraction limits [17].

Possibilities of modern acoustooptic modulators allow to form matrix of 512x512 patterns in photorefractive storage medium. It gives $6{,}8 \cdot 10^{10}$ cross interconnections and $6{,}8 \cdot 10^{10}$ op/s performance with 10 Gb/s bandwidth of an optical channel.

IV. Conclusion

Proposed mathematical model of interconnection pattern in acoustooptic computing devises with dynamically changed architecture can be used as a base for the synthesis of structurally complex memory blocks in optoelectronic processors and can provide level of system performance up to 10^{21} operations per second for existing elements.

Simulations of electromagnetic problem of light wave diffraction on a synthesized grating were described in this paper. It was shown that there is a possibility of reconfiguration of volume interconnections in optoelectronic data processors.

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