NATIONAL ACADEMY OF SCIENCES OF UKRAINE
INSTITUTE OF APPLIED MATHEMATICS AND MECHANICS

WORKSHOP

"CONTEMPORARY ANALYSIS AND NONLINEAR BOUNDARY PROBLEMS"

Sloviansk, October 17-18, 2018

Information on the workshop: The workshop "*Contemporary Analysis and Nonlinear Boundary Problems*" is dedicated to the 80th anniversary of B.V. Bazaliy (1938-2012) and to the centennial anniversary of the National Academy of Sciences of Ukraine.

Bazaliy Borys Vasyliovych was a Ukrainian mathematician, Professor at the Institute of Applied Mathematics and Mechanics of NASU. His most influential contributions are in the area of free boundary problems, nonlinear PDE, and their applications to Mathematical Biology.

The workshop aims to review and discuss the latest trends in Free Boundary Problems, PDE and Analysis.

Organizing Committee:

Yaroslaw Bazaliy (USA), Yevgeniia Yevgenieva (Ukraine), Mykola Krasnoschok (Ukraine), Vasylyeva Nataliya (Ukraine)

Dates: 17-18 October, 2018.

Location: IAMM NASU, Sloviansk, Ukraine.

Approved for publication of the Scientific Council of the Institute of Applied Mathematics and Mechanics of NASU (protocol n. 9 from September 13, 2018)

Yu.S. GORBAN

On weak solutions to nonlinear elliptic degenerate anisotropic equations with L^1-data

Vasyl' Stus Donetsk National University, Vinnytsia, Ukraine
E-mail: yuliyagorban77@gmail.com

We consider the Dirichlet problem

$$-\sum_{i=1}^{n}\left(\nu_i(x)|u_{x_i}|^{q_i-2}u_{x_i}\right)_{x_i} = F(x,u) \ \text{ in } \Omega, \quad u=0 \ \text{ on } \partial\Omega, \tag{1}$$

where Ω is a bounded domain in $\mathbb{R}^n$ with a smooth boundary $\partial\Omega$, $n \geqslant 2$, $q_i \in (1,n)$, $\nu_i : \Omega \to \mathbb{R}$, $\nu_i > 0$ a.e. in Ω, $\nu_i \in L^1_{\mathrm{loc}}(\Omega)$, $\nu_i^{-1/(q_i-1)} \in L^1(\Omega)$, $F : \Omega \times \mathbb{R} \to \mathbb{R}$ is a Carathéodory function.

Definition. *The function $u \in \mathring{W}^{1,1}(\Omega)$ is a weak solution of problem (1) if the following conditions hold:*

i) $F(x,u) \in L^1(\Omega)$;

ii) $\nu_i|u_{x_i}|^{q_i-2}u_{x_i} \in L^1(\Omega)$;

iii) *for every function $w \in C_0^1(\Omega)$ the equality holds*

$$\int_\Omega \left\{\sum_{i=1}^{n}\nu_i|u_{x_i}|^{q_i-2}u_{x_i}w_{x_i}\right\}dx = \int_\Omega F(x,u)w\,dx.$$

Let
$\overline{q} = n\left(\sum_{i=1}^{n} 1/q_i\right)^{-1}$,
and $p_m = n\left(\sum_{i=1}^{n}(1+m_i)/(m_iq_i) - 1\right)^{-1}$ for every $m \in \mathbb{R}^n$, $m_i > 0$.

Theorem. *Let the following conditions hold*

(i) $F(x,\cdot)$ *is nonincreasing on* $\mathbb{R}$ *for a.e.* $x \in \Omega$,

(ii) $F(\cdot,s) \in L^1(\Omega)$ *for any* $s \in \mathbb{R}$,

(iii) $\exists\, m,\sigma \in \mathbb{R}^n$, $m_i > 0$, $\sigma_i > 0 : \overline{q}/(p_m(\overline{q}-1)) < q_i - 1 - 1/m_i$,

$$1/\nu_i \in L^{m_i}(\Omega);\ 1/\sigma_i < 1 - ((q_i-1)\overline{q})/(p_m(\overline{q}-1)),\ \nu_i \in L^{\sigma_i}(\Omega).$$

Then there exists a weak solution of problem (1).

This result continues researches in [1, 2, 3].

[1] A.A. Kovalevsky, Yu.S. Gorban, *On T-solutions of degenerate anisotropic elliptic variational inequalities with L^1-data*, Izv. Math., **75**(1), (2011), 101–160 (in Russian).
[2] A.A. Kovalevsky, Yu.S. Gorban, *Solvability of degenerate anisotropic elliptic second-order equations with L^1-data,* Electron. J. Differential Equations, **167**, (2013), 1–17.
[3] Yu.S. Gorban, *Existence of entropy solutions for nonlinear elliptic degenerate anisotropic equations,* Open Mathematics, **15**, (2017), 768–786.